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Book Review

Topology and Dynamics of Chaos: In Celebration of Robert Gilmore's 70th BirthdayWorld Scientific Series on Nonlinear Science by C. Letellier and R. Gilmore (eds.) Ser. A, vol. 84, World Scientific, 2013;
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This book commemorates the 70th year of the birth of Professor Robert Gilmore, a well-known scientist in the world scientific community. In his honor, an international workshop entitled "From Laser Dynamics to Topology of Chaos" was held June 28–30, 2011, at the Université de Rouen (University of Rouen, France). The topics of the workshop were chosen to cover Gilmore's broad range of scientific interests: laser physics, non-linear dynamics, dynamical systems theory, chaos theory, topological dynamics, and group theory, among others. Prof. Gilmore touched various hot issues, always contributing an original, innovative, and personal perspective.

This book promotes the use of topological dynamics tools to understand, model, and predict the complex "chaotic" behavior in dynamical systems. It consists of three parts. After the preface and introductory chapter, the first section includes four lectures dedicated to the "Emergence of a Chaos Theory". The second section, five lectures in total, gives a concise overview on the "Development of the Topology of Chaos". The third and last section of the book, comprising six lectures, deals with the "Applications of Chaos Theory."

The main subject of the C. Letellier and R. Gilmore introductory chapter is the presentation of some of the basic ideas from the field of nonlinear dynamics and chaos. The terminological inconsistencies in denoting complex behaviors (turbulent, aperiodic, chaotic, strange, etc.), are briefly discussed. Next, the topological analysis program and methods are sketched and illustrated using the chaotic attractor solution to the famous Sprott D system (i.e., a non-hyperbolic chaotic system with a single but unstable equilibrium—see also X. Wang, G. Chen, *A chaotic system with only one stable equilibrium*. Commun. Nonlinear Sci. Numer. Simulat. Vol. 17, 3, 2012, 1264–1272). Some remarks regarding the construction of a representation theory of dynamical systems close the chapter.

The lecture by R. Abraham presents the main historical steps in the acquisition of knowledge in physics and mathematics that led to dynamical systems and chaos theories. Unfortunately, I found that at least two famous scientists were not mentioned: A. N. Kolmogorov (1903–1987) and E. Hopf (1902–1983).

The next lecture, by C. Mira, describes the scientific activity of the Toulouse research group between 1958 and 1976 in complex nonlinear dynamics. In fact, this text is dedicated to Prof. I. Gumowski, who provided an enormous contribution to nonlinear science, and without whom the Toulouse exploratory group would not exist. I would point only to the concept of chaotic images, i.e., the Gumowski-Mira map, which is a two-dimensional recurrence relation whose iterations lead to a variety of 'lively' patterns [or "aesthetic" attractors—see map (1) on p. 53].

Before the era of fast computers, solving a nonlinear dynamical system required subtle and nontrivial mathematical techniques, and could only be accomplished for a selected class of dynamical

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systems. The lecture by R. Lozi is concerned with a discussion of the validity and relevance of numerical solutions. The first section concentrates on the possibility of questionable computations. Here, the crucial role of the Poincaré map is explored, helping to illustrate the close relationship between continuous and discrete dynamical systems models. Next, the material is analysed and annotated using spectacular examples—for the discrete case, for example, a modified Hénon map and logistic and tent maps in 1D, and a Hénon and Lozi map in 2D. The results are complemented by carefully chosen continuous models, including Lorenz, Rössler, and Chua systems. The significance of the so-called numerical turbulence is also outlined, i.e., the mathematical phenomenon that occurs due to incorrectness of a calculation procedure stipulated by discreteness.

The lecture authored by O. E. Rössler and C. Letellier focused on "chemical chaos." The significant contribution of Prof. Rössler in the field of chemical and biochemical reactions and dynamical systems is well known, and it began in 1975. Some of the most important steps leading to modern development of the subject are presented first. The source of early inspirations is indicated, including the paper by Khaikin (1930), the Lorenz paper (1963) and the Li & Yorke paper (1975). His earliest paper on chaos [Phys. Lett. A, 57, 397–398 (1976)] is presented with commentaries.

The next two lectures deal with elements of the knot theory. J. S. Birman presents an excellent introduction to the concept of Lorenz knots. What is a knot? Formally speaking, a knot K is a globally injective embedding e of the sphere S^1 in the Euclidean R^3 space, i.e., e: $S^1 \rightarrow \mathrm{R}^3$; that is, $e(S^1) = K \subset \mathrm{R}^3$. A brief history of knot theory is presented first. Next, the special classes of knots and links—namely, torus knots and links—are described. The Lorenz knots associated with the famous Lorenz system are, in fact, the closed periodic orbits of the Lorenz model. In particular, the important conclusion that all torus knots are Lorenz knots is elucidated. The three methods of parameterization of Lorenz knots are described. The consequences of these parameterizations are illustrated, together with an explanation that the class of Lorenz knots coincides with the class of modular knots.

The lecture by M. Natiello and H. Solari is devoted to the relationship between braid theory and the organisation of periodic orbits of 3D dynamical systems. An algorithm for estimating the minimal periodic orbit structure of systems whose Poincaré section is homotopic to a disc is described and explained. The extension of this approach to other class dynamical systems is also presented.

The lecture by R. Gilmore concerns the topological methods for analyzing chaotic data. First, the limitations of previous tools such as fractal dimension, Lyapunov exponents, and spectra of Lyapunov exponents are mentioned. The following brief discussion presents a review of topological indicators that determine the stretching and squeezing processes acting on flows in phase space that are responsible for generating chaotic data. In this context, the famous Birman–Williams theorem is discussed, which states that all periodic orbits can be isotoped down to a two-dimensional branched manifold, preserving their topological structure. The notion of the branched manifold is explained, and four branched manifolds connected with the Lorenz, Rössler, Duffing, and van der Pol dynamical systems are sketched. A reflection regarding the extension of the topological analysis program to higher spatial dimensions finishes the lecture.

This theme is continued in the lecture by M. Lefranc, which outlines the topological analysis program in higher-dimensional spaces. Two preliminary possibilities are described: first, the use of catastrophe theory methods in the analysis of higher-dimensional dynamical systems by describing experimental signatures of cusps in weakly coupled chaotic models; and second, a triangulation-based formalism providing a novel understanding of the principle of determinism, in which this rule is enforced through the construction of orientation-preserving dynamics on triangulated surfaces.

The lecture by C. Letellier presents results regarding the relation of cover and image dynamical systems. A methodical approach developed by the author and R. Gilmore for constructing dynamical systems with a specific symmetry group is briefly described. Two particular cases of images are discussed: the image for which the rotation axis crosses the attractor, and the toroidal attractor connected with the Li dynamical system. The NARMAX model for sunspot data is briefly analysed.

The third section of the book, “Good artist copy, great artist steal,” contains five lectures inspired by the work of Prof. R. Gilmore in topological dynamics. The lecture by N. Tufillaro focuses on ocean color, the notion of the spectral study of light as it is scattered about the water column. Two remote-sensing ocean imagers, MERIS and HICO, are briefly described. The applications of derivative spectroscopy to complex coastal waters (such as the San Francisco Bay) are presented and discussed. The lecture by G. B. Mandlin summarizes recent studies on the dynamics of a set of forced coupled excitable units. Another lecture, by J-M. Malasoma, considers conditions sufficient for the existence of a class of minimal smooth chaotic flows in the “space” on 3D autonomous quadratic differential systems. The lecture by C. Gilmore addresses the close returns method applied to test for the presence of chaotic behavior in selected financial time-series data. The close returns test is a topological-based procedure consisting of a qualitative component (graphical test) and a quantitative factor (null hypothesis that the data are independently and identically distributed). The results indicate that there is no evidence of the chaotic regime.

The lecture by K. Gilmore and R. Gilmore is devoted to an analysis of dynamical systems connected with spin-torque nano-oscillators. These objects are described by the Landau–Lifshitz–Slonczewski equation, which forms a set of ordinary differential equations whose source terms are polynomials of a degree no greater than 2. The lecture provides a very interesting link between spintronics and chaos theory from a dynamical systems theory point of view.

The last chapter, by H. G. Solari, presents the academic and scientific activities within the wider context of the life work of Prof. Gilmore (see also: http://www.physics.drexel.edu/~bob/).

In sum, this book is an excellent work, as all authors had an exciting and interesting story to tell—the progress and mathematical foundations of a very topical problem in nonlinear science: topological analysis of chaotic dynamics. It is therefore a must-read for those interested in understanding the unexpected and indirect connections that bind topology, ordinary differential equations, and dynamical systems theory.